WINDOWS USED FOR IMPACT TESTING

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ABSTRACT. *Windowing is a common signal processing technique to reduce measurement errors. This paper reviews the force and exponential windows, which are used for impact testing. In general, the exponential window is applied the response signals to reduce leakage and the force window is applied to the impact signal to reduce noise. The need for and proper use of these windows are explained, and guidelines for selecting the properties of the windows are discussed. Also mentioned are the occasions in which the windows should not be applied. The reasons that the exponential window must also be applied to the force signal are outlined and the damping correction for the exponential window is derived.*

NOMENCLATURE

A_r	residue of mode r
e^t, e^s	exponential function of t, exponential function of s,
$f(t)$	input time signal
$F(s)$	Laplace transform of $f(t)$
$h(t)$	system impulse response function
$H(s)$	system transfer function (s-domain)
s	Laplace domain independent variable
t	time domain independent variable
t_o	pretrigger delay time
T	time record length
$w_e(t)$	exponential window time function
$x(t)$	response time signal
$X(s)$	Laplace transform of $x(t)$
β	reciprocal of exponential window time constant (rad/sec)
λ_r	pole of mode r
σ_r	damping factor of mode r
ω_r	damped natural frequency of mode r
Ω_r	undamped natural frequency of mode r
τ	exponential window time constant (sec)
ζ_r	damping ratio of mode r
$y(t) \Leftrightarrow Y(s)$	denotes a Laplace transform pair
FRF	frequency response function
IRF	impulse response function

1. Introduction

Windowing is a common signal processing technique available on all modem data acquisition systems used for modal testing. The force and exponential windows arc time domain weighting functions that have been developed for the transient signals of impact testing.“’ These windows are designed to reduce the effects of leakage and improve the signal-to-noise ratio of the measured signals. In general, to improve impact testing FRF measurements, the force and exponential windows should almost always be applied to the time signals, with very few exceptions.

To correctly measure a frequency response function (FRF), the exponential window must also be applied to the input signal, in addition to the force window, if it is applied to the response signals. The effect of the exponential window is to increase the apparent damping of the measured system. This effect, which is governed by the shift property of the Laplace transform, is predictable and easily corrected in the estimated modal parameters.

2. Causer and Symptoms of Leakage in Impact Testing

Leakage is a signal processing bias error caused hy the violating an assumption of the discrete Fourier transform. The assumption is that the signal to be transformed is periodic within the sample period. For cases in which both the input and output are harmonic functions of the sampling period or arc completely observed transients, there will be no leakage errors. Since real systems do not generally respond as multiples of some arbitrary sampling frequency, windows are used to constrain the signal to more closely meet the requirement of a completely observed transient. The effects of leakage on measured FRFs is an underestimation of the magnitude at the peaks and distortion of the phase, and a drop in coherence at the peaks,“’

For a signal to be a completely observed transient, it must start at zero and end at zero within the sampling period. For the transient signals of impact testing, the input is always completely observable, but the response may not he. If the response does not decay to near zero by the end of the sampling period, the exponential window is applied to reduce leakage. Note that windowing can only reduce the effects of leakage, not entirely eliminate them.

3. Use of the Exponential Window

The exponential window is simply an exponential function as defined by either of the two forms in equation (I), where the parameter τ is the time constant of the exponential function. The time variable for the exponential function struts at zero, regardless if a pretrigger delay is used in the measurement.

$$w_e(t) = e^{-t/\tau} \quad \text{or} \quad w_e(t) = e^{-\beta t} \qquad \text{where } \beta = \frac{1}{\tau} \qquad (1)$$

The principal purpose of the exponential window is to reduce the effects of leakage on lightly damped response signals. The transient response of a lightly damped system will typically not decay to near zero by the end of the time record, as show in Figure I(a). For lightly damped systems, the exponential window should reduce the measured response signal at the end of the time record to approximately one percent, as shown in Figure 1(b), to effectively reduce the effects of leakage.

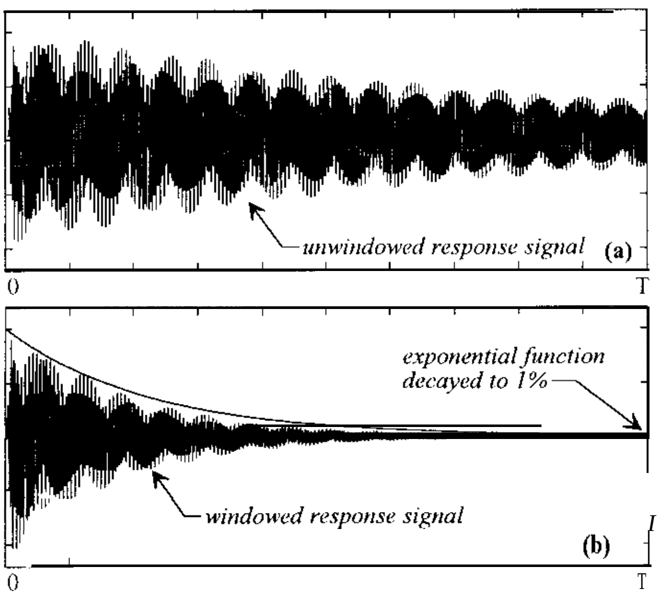

Figure 1. (a) *Unwindowed and* (b) *windowed response signal of a lightly damped system and the exponential window.*

The exponential window can also improve the signal-to-noise ratio of heavily damped response signals. It may seem that an exponential window is not needed since the transient response decays very quickly, within the measured time record, and the signal is a completely observed transient. However, an exponential window is used in this case to attenuate the noise on the measured output after the response has decayed due to system damping. For heavily damped systems, the exponential window should follow the damping of the system, as shown in Figure 2.

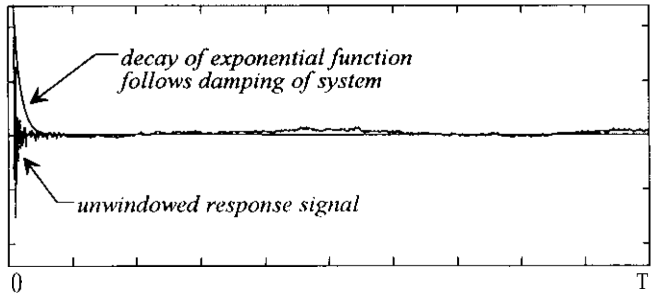

Figure 2. *Unwindowed response signal of a heavily damped system and the exponential window.*

The decay of the exponential window is typically defined in commercially available data acquisition software by one of the several ways listed below:

- specifying the time constant (τ) in seconds
- specifying the reciprocal parameter(B) in rad/sec
- specifying the value of the exponential function at the end of the time period as a percentage of unity
- specifying the value of the exponential function, as a percentage of unity, at some point on the time axis, as a percentage of the time record length
- graphically shaping the exponential curve with the mouse

A common suggestion is that the time constant should bc one quarter of the time record length, which creates an exponential function that decays to about two percent at the end of the time record. However, this would not he appropriate for heavily damped systems, as explained above. Also, note that if the frequency span; number of spectral lines, or any other measurement parameter that affects the time record length is changed, then τ or β must also be updated so that the shape of the exponential window is prcscrved.

4. Use of the Force Window

The purpose of the force window is to improve the signal-to-noise ratio of the measured input by eliminating the noise on the signal following the duration of the impact. After the impact, the impactor is no longer in contact with the surface and can not impart any excitation into the system The data in the trailing segment of the unfiltered force signal consists of only electrical noise on the input channel. However, due to the short duration of the impulse, the total energy of the noise may be on the same order as the energy of the input force. The force window passes the initial segment of the time record containing the impact signal and suppresses the noise in the remainder of the time record. The trailing segment of the filtered force signal also contains response to the anti-aliasing filter, which should not he truncated by the force window because it contains energy of the input.

The force window is unity over the leading five to ten percent of the time record, has a steep cosine taper to zero, and is zero for the remainder of the time record. In some cases, the trailing segment of the time record is set to the average value of the noise instead of zero. As will be discussed in a following section, the exponential window must also be applied to the force signal, in addition to the force window. The force window and the combination for the force and exponential windows are shown in Figure 3.

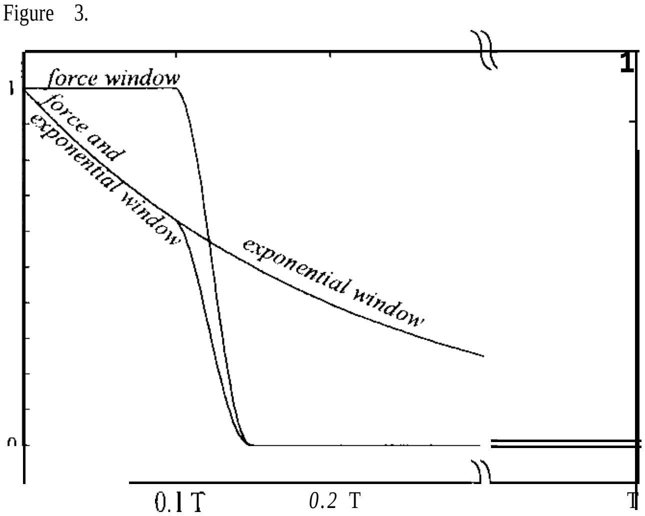

Figure 3. *The force window.*

The force window is defined by the duration of the leading unity portion, which is commonly referred to as the "length" of the window In commercially available data acquisition software, the force window length is typically specified in either absolute time or as a percentage of the time record length. A duration of 5.10% of the time record length, after the trigger, is usually recommended for the length of the force window. Note that the force window length may or may not by default take into consideration the use of a pretrigger.

5. Exception to the Rule

The force and exponential windows should normally be applied to the time signals when using impact testing. The exception to this rule is when the measured signals contain significant components of periodic noise. In this case, the windows will smear the periodic noise components, contaminating the adjacent spectral lines. The periodic noise components are undamped and, if not windowed, appear in the spectrum as narrow peaks. However, the exponential window increases the apparent damping of the measured signals, which results in broadening of the periodic noise peaks. The line shapes of the force and exponential windows arc show in Figure 4. The line shape of a time domain window determines the properties of the window in the frequency domain

The types of periodic noise commonly encountered with impact testing include the DC-component, electrical line noise, and periodic excitation sources. Because of the frequency domain effects of the windows, the periodic noise must be removed from the data before applying the windows in the time domain. If possible, the noise sources should be eliminated by appropriate measurement practices. If the source of the noise can not be eliminated, then the periodic noise components can be removed by signal processing techniques described Ref. [3].

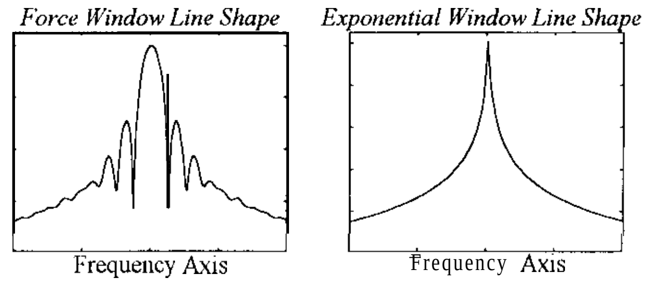

Figure 4. *Line shapes of the force and exponential windows.*

6. Applying the Exponential Window to the Force Signal

Proper use of the exponential window requires that it must be applied to the both response signals and the input signal. Although an FRF is defined as a division of the output spectrum by the input spectrum, applying the exponential window to both the input and output does not cancel the effect of the window from the FRF

The shift properties of the Laplace transform"' govern the effects of the exponential window and illustrate why it must be applied to both the input and output time signals. In the discussion below, the Laplace variable (s) is used, but the Fourier transform is equivalent to the Laplace transform evaluated at the imaginary (jo) axis.

Multiplying a time signal $y(t)$ by an exponential function shifts the independent variable of the associated Laplace transform $Y(s)$.

$$\text{If} \quad y(t) \Leftrightarrow Y(s), \qquad \text{then} \qquad e^{at}y(t) = Y(s \quad a) \tag{2}$$

Shifting the independent variable of a time signal $y(t)$ by an amount I, multiplies the associated Laplace tmnsfonn $Y(s)$ by an exponential function in the s-domain.

$$\text{If} \quad y(t) \Leftrightarrow Y(s), \qquad \text{then} \qquad y(t - t_o) \Leftrightarrow e^{-t_o s}Y(s) \tag{3}$$

Equation (2) governs the effect of the exponential window and equation (3) governs the effect of the pretrigger delay.

The unwindowed, equation (4), and exponentially windowed, equation (5), input $f(t)$ and output x(t) time signals are transformed to the s-domain

$$f(t) \Leftrightarrow F(s) \qquad \text{and} \qquad x(t) \Leftrightarrow X(s) \tag{4}$$

$$e^{-\beta t}f(t) \Leftrightarrow F(s+\beta) \qquad \text{and} \qquad e^{-\beta t}x(t) \Leftrightarrow X(s+\beta) \tag{5}$$

The transfer function, *H(s)*, is defined as the ratio of the Laplace transforms of the output and input. For the case in which neither the output or input time signals are exponentially windowed,

$$H(s) = \frac{X(s)}{F(s)} \tag{6}$$

For the case in which both the output and input signals are exponentially windowed,

$$H(s+\beta) = \frac{X(s+\beta)}{F(s+\beta)} \tag{7}$$

Since the transfer function is computed from the shifted input and output functions. it is a function of the same independent variable. For the case in which the output signal is exponentially windowed but the input signal is not,

$$H(?) = \frac{X(s+\beta)}{F(s)} \tag{8}$$

Equation (7) clearly shows that the exponential window must be applied to both the response and force time signals so that the independent variable of the transfer functiou is unambiguously defined. If the exponential window is applied to the response, its transform is a function of $s+\beta$, but if the window is not applied the force, its transform is a function of just s, as indicated in equation (8). So the question is, what is the independent variable of H for this case'? The practical consequences of applying the exponential window to an impact signal are investigated in Ref. [5] by considering the dependence of F on β. Also considered are the more subtle, combined effects of the exponential window and a pretrigger delay. The conclusions can be summarized as follows.

The exponential window and pretrigger delay affect the measured FRF and estimated modal parameters in a predictable manner. The effects are best understood by studying the transfer function of the system and the Laplace transform of a pulse input function. It is the combination of the Laplace transform shin properties which govern the effects of the exponential window and the pretrigger delay that necessitate the exponential window always be applied to the force signal when measuring frequency response functions.

'The exponential window effect of increasing the apparent damping of the measured response is widely acknowledged and readily corrected in the modal parameters. The exponential window essentially shifts the plane of the s-domain input and output functions that are actually measured, and to measure the correct FRF, the sane planes of the input and output functions must he measured. That is, both the input and output must by a function of $s+\beta$. The errors introduced by not windowing the force signal will not affect the estimated frequency or damping hut can effect the scaling of the residue due to au incorrect FRF magnitude, especially if a pretrigger delay is used in the measurements.

Pretriggering is commonly used in impact testing to observe the leading edge of the measured time signals, and its effects are also governed by a shift property of the Laplace transform. The errors introduced by not windowing the force signal are amplified when a pretrigger delay is used in the measurements, which can lead to a significant underestimation of the residue. Although the residues could he corrected for this effect, this is an unnecessary complication that can he avoided by applying the exponential window to the force signal.

7. Correction for the Exponential Window

The exponential window increases the apparent damping of the measured system, and the amount of added damping is determined by the exponential time constant. This effect is predictable and the correction for estimated modal parameters is developed below.[6] Stating with equation (7), the FRF is inverse transformed to the impulse response function (IRF), where $h(t)$ is the IRF of the true (i.e., unwindowed) system.

$$H(s+\beta) \Leftrightarrow e^{-\beta t}h(t) \tag{9}$$

The IRF can he written as a summation of damped exponential terms, where λ_r is the complex eigenvalue of mode r, A_r is the residue for mode r, and the caret notation (^) denotes a parameter associated with the measured (i.e., windowed) system

$$e^{-\beta t}h(t) = \hat{h}(t) \quad \text{or} \quad e^{-\beta t}\sum_r e^{\lambda_r t}A_r = \sum_r e^{\hat{\lambda}_r t}\hat{A} \tag{10}$$

Equating the like terms in equation (10) and inserting the real and imaginary parts of the eigenvalues yields the correction for the exponential window. where ω_r and σ_r are the damped natural frequency and damping factor of mode r, respectively.

$$e^{-\beta t}e^{\lambda_r t} = e^{\hat{\lambda}_r t} \quad \text{and} \quad A_r = \hat{A}_r \tag{11}$$

$$-\beta + \sigma_r + j\omega_r = \hat{\sigma}_r + j\hat{\omega}_r \tag{12}$$

$$\sigma_r = \hat{\sigma}_r + \beta \quad \text{and} \quad \omega_r = \hat{\omega}_r \tag{13}$$

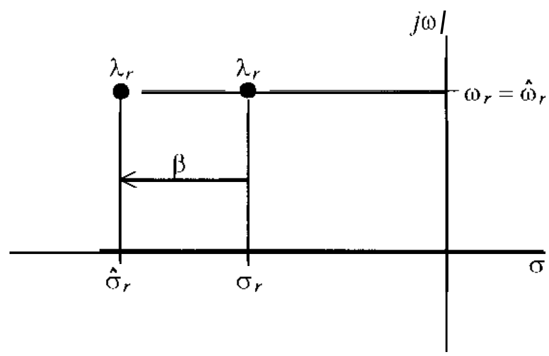

Figure 5. *The exponential window effects and correction on the complex plane.*

Equations 11 and 13 indicate that the damped natural frequencies and residues of the measured system are identical to those of the true system and the difference of the damping factors between the true and measured systems is a function of the exponential window time constant. The effects of the exponential wiudow described in the above equations are illustrated on the complex plane in Figure 5. Since the damping factors of natural system are expected to be negative, the damping factors of the measured system will have larger negative values than that of the true system, which causes the apparent increased damping in the measurements. Note that since the measured FRFs are computed from data modified by the exponential window, residue and modal scaling calculations and FRF synthesis should use the uncorrected poles.

A damping correction for the exponential window historically given in the literature[7,8,9] that deals with the damping ratio of a mode, ζ_r

$$\zeta_r = \hat{\zeta}_r - \frac{\beta}{\Omega_r}\ , \quad \text{where} \quad \zeta_r = \frac{\sigma_r}{\Omega_r} \quad \text{and} \quad \Omega_r = \sqrt{\sigma_r^2 + \omega_r^2} \qquad (14)$$

is actually an approximate method.[10] The approximation is that the estimated undamped natural frequency, $\hat{\Omega}_r$, is equal to the true undamped natural frequency, Ω_r, when in fact $\hat{\Omega}_r > \Omega_r$ since $|\hat{\sigma}_r| > |\sigma_r|$. The approximate correction underestimates the damping ratio by a factor of $\Omega_r/\hat{\Omega}_r$, but approaches the true value asymptotically as the damped natural frequency increases, for a given damping ratio and exponential time constant.

Another possible effect of the exponential window which should be noted is that it may complicate separation of closely spaced modes due to the increased damping. Figure 6 shows the response to an impact of an steel T-plate (approx. 18x29x29cm, 0.635cm thick) and the applied exponential windows. The windows decay to 1% at 100%, 50%, 20% and 10% of the time record length. which corresponds to a time constant of 0.2171, 0.1086, 0.0434 and 0.0217 sec., respectively. Figure 7 shows the FRF for each of these four cases, note the two closely spaced modes at 580 and 588 Hz. As the added damping from the expo- nential window increases, these two peaks merge such that two distinct modes cannot be distinguished.

8. References

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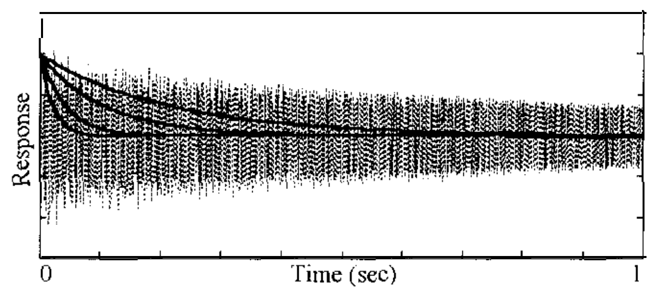

Figure 6. *Transient response of a lightly damped system and applied exponential windows.*

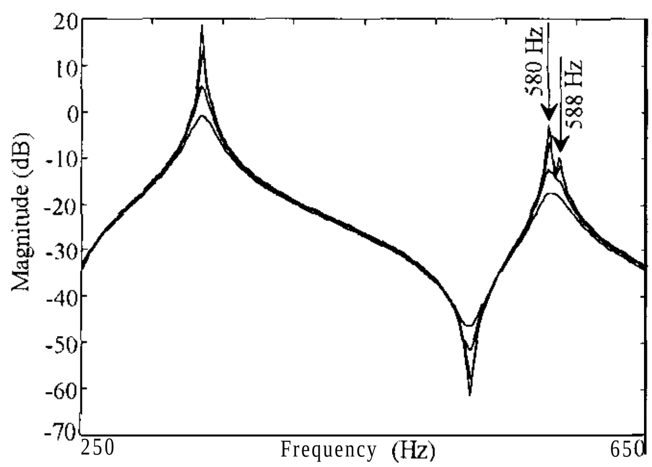

Figure 7. *FRFs of a lightly damped system showing the increase in damping from the applied exponential windows.*