1. INTRODUCTION

Today's engineers are faced with many complex noise and vibration problems associated with the design and troubleshooting of structures. Never before have structures had so many constraints, such as legislative, cost, and durability, imposed upon them. Because of these constraints, present day structures are typically more complex in terms of design and materials. Therefore, the engineering that goes into these structures must be more exact than ever before. The structural analyst must design with accurate and realistic models and the experimentalist must be able to accurately define a structure's dynamic response to specified input forces.

Recently, tools have been developed to assist the structural engineer in these areas. The analyst now uses sophisticated finite element programs, such as Nastran, to aid in the understanding and the design of structures. The experimentalist has sophisticated digital signal analysis equipment that aids him in quantifying and understanding a structure's dynamic and input forces. However, there is little communication between the analyst and experimentalist. Due to the complexity of todays problems, the need for communication and exchange of ideas is even more critical.

The major reason for this lack of communication is that the analytical and experimental engineers do not understand each other's terminology and methods of problem solution. The discussion that will follow will attempt to fill this void.

The major point of the text is to show how frequency response function (FRF) measurements are related to the structure's mode shapes and vibrational frequencies. This overall objective will be accomplished by building a mathematical foundation from the analytical and experimental point of view.

The material is divided into two sections: single degree of freedom, and multiple degree of freedom systems. The single degree of freedom system will be used as a means for defining some standard terminology. The multiple degree of freedom material is further divided into: undamped, proportionally damped, and non-proportionally damped systems. Throughout the discussion emphasis will be placed on the relationship of frequency response measurements to modal vectors. Furthermore, the following concepts will be presented:

- Modal Frequencies, Eigenvalues, System Poles, Characteristic Roots (λ_r)
- Modal Vectors, Eigenvectors ($\{\psi_r\}$, $\{\phi_r\}$)
- Modal Coefficients (ψ_{pr}), Residues (A_{pqr})
- Real Modal Vectors, Complex Modal Vectors
- Physical Coordinates, Generalized Coordinates
- Principal Coordinates, Modal coordinates
- Modal Mass, Damping, and Stiffness (M_r, C_r, K_r)
- Modal A, Modal B (M_{A_r}, M_{B_r})

1.1 Degrees of Freedom

The development of any theoretical concept in the area of vibrations, including modal analysis, depends upon an understanding of the concept of the number of degrees of freedom (N) of a system. This concept is extremely important to the area of modal analysis since the number of modes of vibration of a mechanical system is equal to the number of degrees of freedom. From a practical point of view, the relationship between this theoretical definition of the number of degrees of freedom and the number of measurement degrees of freedom (N_o, N_i) is often confusing. For this reason, the concept of degree of freedom will be reviewed as a preliminary to the following modal analysis material.

To begin with, the basic definition that is normally associated with the concept of the number of degrees of freedom involves the following statement: The number of degrees of freedom for a mechanical system is equal to the number of independent coordinates (or minimum number of coordinates) that is required to locate and orient each mass in the mechanical system at any instant in time. As this definition is applied to a point mass, three degrees of freedom are required since the location of the point mass involves knowing the x, y, and z translations of the center of gravity of the point mass. As this definition is applied to a rigid body mass, six degrees of freedom are required since θ_x, θ_y, and θ_z rotations are required in addition to the x, y, and z translations in order to define both the orientation and location of the rigid body mass at any instant in time. This concept is represented in Figure 1-1. As this definition is extended to any general deformable body, it should be obvious that the number of degrees of freedom can now be considered as infinite. While this is theoretically true, it is quite common, particularly with respect to finite element methods, to view the general deformable body in terms of a large number of physical points of interest with six degrees of freedom for each of the physical points. In this way, the infinite number of degrees of freedom can be reduced to a large but finite number.

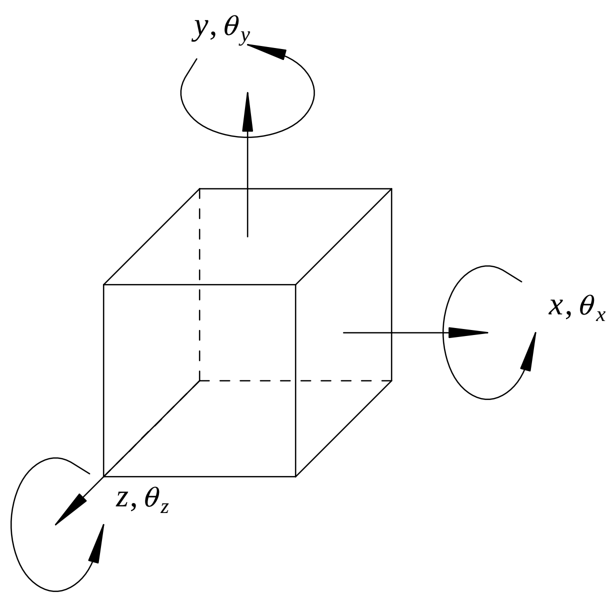

Figure 1-1. Degrees of Freedom of a Rigid Body

When measurement limitations are imposed upon this theoretical concept of the number of degrees of freedom of a mechanical system, the difference between the theoretical number of degrees of freedom (N) and the number of measurement degrees of freedom (N_o, N_i) begins to evolve. Initially, for a general deformable body, the number of degrees of freedom (N) can be considered to be infinite or equal to some large finite number if a limited set of physical points of interest is considered as discussed in the previous paragraph. The first measurement limitation that needs to be considered is that there is normally a limited frequency range that is of interest to the analysis. For example, most dominant structural modes of vibration for an automobile would be located between 0 and 200 Hertz. As this limitation is considered, the number of degrees of freedom of this system that are of interest is now reduced from infinity to a reasonable finite number. The next measurement limitation that needs to be considered involves the physical limitation of the measurement system in terms of amplitude. A common limitation of transducers, signal conditioning and data acquisition systems results in a dynamic range of 80 to 100 db (10^4 to 10^5) in the measurement. This means that the number of degrees of freedom is reduced further due to the dynamic range limitations of the measurement instrumentation. Finally, since few rotational transducers exist at this time, the normal measurements that are made involve only translational quantities (displacement, velocity, acceleration, force) and thus do not include rotational effects, or RDOF. In summary, even for the general deformable body, the theoretical number of degrees of freedom that are of interest are limited to a very reasonable finite value ($N = 1 - 50$). Therefore, this number of degrees of freedom (N) is the number of modes of vibration that are of interest.

Finally, then, the number of measurement degrees of freedom (N_o, N_i) can be defined as the number of physical locations at which measurements are made times the number of measurements made at each physical location. For example, if x, y, and z accelerations are measured at each of 100 physical locations on a general deformable body, the number of measurement degrees of freedom would be equal to 300. It should be obvious that since the physical locations are chosen somewhat arbitrarily, and certainly without exact knowledge of the modes of vibration that are of interest, that there is no specific relationship between the number of degrees of freedom (N) and the number of measurement degrees of freedom (N_o, N_i). In general, in order to define N modes of vibration of a mechanical system, N_o, N_i must be equal to or larger than N. Note also that even though N_o, N_i is larger than N, this is not a guarantee that N modes of vibration can be found from N_o, N_i measurement degrees of freedom. The N_o, N_i measurement degrees of freedom must include physical locations that allow a unique determination of the N modes of vibration. For example, if none of the measurement degrees of freedom are located on a portion of the mechanical system that is active in one of the N modes of vibration, portions of the modal parameters for this mode of vibration can not be found.

In the development of the single and multiple degree of freedom information in the following Sections, the assumption is made that a set of N measurement degrees of freedom ($N_o, N_i = N$) exist that will allow for N modes of vibration to be determined. In reality, N_o, N_i is always chosen much larger than N since a prior knowledge of the modes of vibration is not available. If the set of N_o, N_i measurement degrees of freedom is large enough and if the N_o, N_i measurement degrees of freedom are distributed uniformly over the general deformable body, the N modes of vibration will normally be found.

1.2 Basic Assumptions

Before proceeding, the basic assumptions must be established before the theory can be developed. The first assumption is that the structure is a linear system whose dynamics may be represented by a set of linear, second order, differential equations. The second assumption is that the structure during the test can be considered as time invariant. This assumption implies that the coefficients in the linear, second order, differential equations are constants and do not vary with time. The third assumption is that the structure is observable. While this may seem trivial, this means that the system characteristics that are affecting the dynamics can be measured and that there are sufficient sensors to adequately describe the input-output characteristics of the system. Another assumption that is often made is that the structure obeys Maxwell's reciprocity theorem. Maxwell's reciprocity theorem, in terms of frequency response function measurements, implies the following: if one measures the frequency response function between points p and q by exciting at p and measuring the response at q, the same frequency response function will be measured by exciting at q and measuring the response at p $\left(H_{pq} = H_{qp} \right)$.