UC-SDRL-CN-20-263-662

VIBRATIONS:
ANALYTICAL AND EXPERIMENTAL MODAL ANALYSIS

Dr. Randall J. Allemang, Professor
Structural Dynamics Research Laboratory
Department of Mechanical, Industrial and Nuclear Engineering
University of Cincinnati
Cincinnati, Ohio 45221-0072

February 1999

Approved for public release

Copyright © 1990-1999 UC-SDRL
All Rights Reserved

PREFACE

An initial version of the following set of notes was originally prepared by Dave Formenti in 1977 for use in a short course at the University of Cincinnati on the subject of "Experimental Modal Analysis". The notes have since been rewritten several times to standardize the notation and add further clarification and additional topics by several authors including, R. J. Allemang, R. W. Rost, T. J. Severyn, and J. M. Leuridan, for use in other short courses and the dual level Mechanical Vibrations II (20-263-662) course at the University of Cincinnati. Any comments relative to corrections or improvements will be welcomed.

PRINTING/REVISION HISTORY

In an attempt to improve and correct these notes, several revisions have been made. Please be certain that you have the latest revision.

March 1990
Revision 1: February 1992
Revision 2: June 1992
Revision 3: July 1993
Revision 4: March 1994
Revision 5: February 1999

NOMENCLATURE

Matrix Notation

$\{..\}$	braces enclose column vector expressions
$\{..\}^T$	row vector expressions
$[..]$	brackets enclose matrix expressions
$[..]^H$	complex conjugate transpose, or Hermitian transpose, of a matrix
$[..]^T$	transpose of a matrix
$[..]^{-1}$	inverse of a matrix
$[..]^+$	generalized inverse (pseudoinverse)
$[\,..\,]_{q \times p}$	size of a matrix: q rows, p columns
$\lceil\,..\,\rfloor$	diagonal matrix

Operator Notation

A^*	complex conjugate
$\mathbf{F}$	Fourier transform
$\mathbf{F}^{-1}$	inverse Fourier transform
$\mathbf{H}$	Hilbert transform
$\mathbf{H}^{-1}$	inverse Hilbert transform
ln	natural logarithm
$\mathbf{L}$	Laplace transform
$\mathbf{L}^{-1}$	inverse Laplace transform
$\mathrm{Re} + j\,\mathrm{Im}$	complex number: real part "Re", imaginary part "Im"
$\dot{x}$	first derivative with respect to time of dependent variable x
$\ddot{x}$	second derivative with respect to time of dependent variable x
$\bar{y}$	mean value of y
$\hat{y}$	estimated value of y

$\sum_{i=1}^{n} A_i\ B_i$	summation of $A_i\ B_i$ from i = 1 to n
$\frac{\partial}{\partial t}$	partial derivative with respect to independent variable "t"
$\det[\,.\,.\,]$	determinant of a matrix
$\|.\,.\|_2$	Euclidean norm

Roman Alphabet

A_{pqr}	residue for response location p, reference location q, of mode r
C	damping
e	base e (2.71828...)
F	input force
F_q	spectrum of q^{th} reference†
$h(t)$	impulse response function†
$h_{pq}(t)$	impulse response function for response location p, reference location q †
$H(s)$	transfer function†
$H(\omega)$	frequency response function, when no ambiguity exist, H is used instead of $H(\omega)^\dagger$
$H_{pq}(\omega)$	frequency response function for response location p, reference location q, when no ambiguity exist, H_{pq} is used instead of $H_{pq}(\omega)^\dagger$
$[I]$	identity matrix
j	$\sqrt{-1}$
K	stiffness
K_r	modal stiffness for mode r
L	modal participation factor
M	mass
M_r	modal mass for mode r
M_{A_r}	modal A for mode r
M_{B_r}	modal B for mode r

N	number of modes
N_i	number of references (inputs)
N_o	number of responses (outputs)
p	output, or response point (subscript)
q	input, or reference point (subscript)
r	mode number (subscript)
R_I	residual inertia
R_F	residual flexibility
s	Laplace domain variable
t	independent variable of time (sec)
t_k	discrete value of time (sec)
	$t_k = k\,\Delta t$
T	sample period
x	displacement in physical coordinates
X	response
X_p	spectrum of p^{th} response†
z	Z domain variable

Greek Alphabet

$\delta(t)$	Dirac impulse function
Δf	discrete interval of frequency (Hertz or cycles/sec)
Δt	discrete interval of sample time (sec)
ε	small number
η	noise on the output
λ_r	r^{th} complex eigenvalue, or system pole
	$\lambda_r = \sigma_r + j\omega_r$
$\lceil \Lambda \rfloor$	diagonal matrix of poles in Laplace domain
ν	noise on the input
ω	variable of frequency (rad/sec)
ω_r	imaginary part of the system pole, or damped natural frequency, for mode r (rad/sec)
	$\omega_r = \Omega_r \sqrt{1 - \zeta_r^{\,2}}$
Ω_r	undamped natural frequency (rad/sec)
	$\Omega_r = \sqrt{\sigma_r^{\,2} + \omega_r^{\,2}}$
ϕ_{pr}	scaled p^{th} response of normal modal vector for mode r
$\{\phi\}_r$	scaled normal modal vector for mode r
$[\Phi]$	scaled normal modal vector matrix
$\{\psi\}$	scaled eigenvector
ψ_{pr}	scaled p^{th} response of a complex modal vector for mode r
$\{\psi\}_r$	scaled complex modal vector for mode r
$[\Psi]$	scaled complex modal vector matrix
σ	variable of damping (rad/sec)
σ_r	real part of the system pole, or damping factor, for mode r
ζ	damping ratio
ζ_r	damping ratio for mode r

† vector implied by definition of function